J–3283

M. A./M.Sc. (Previous)

Term End Examination, June-July, 2018

MATHEMATICS

Paper Third

(Partial Differential Equation)

Time : Three Hours] [*Maximum Marks : 70*

[*Minimum Pass Marks : 21*

Instructions for Candidate :

Section–A : Question Nos. **01** to **08** are very short answer type questions. Attempt all questions. Each question carries 01 mark. Answer each of these questions in 1 or 2 words/1 sentence.

Section–B : Question Nos. **09** to **14** are half short answer type questions. Attempt any *four* questions. Each question carries $2\frac{1}{2}$ marks. Answer each of these questions in about 75 words or half page.

Section–C : Question Nos. **15** to **18** are short answer type questions. Attempt any *three* questions. Each question carries 05 marks. Answer each of these questions in about **150** words or one page.

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Section–D : Question Nos. **19** to **22** are half long answer type questions. Attempt any *two* questions. Each question carries 10 marks. Answer each of these questions in about **300** words or two pages.

Section–E : Question Nos. **23** and **24** are long answer type questions. Attempt any *one* question. Each question carries 17 marks. Answer each of these questions in about **600—750** words or 04—05 pages.

Section—A

1. Partial differential equation $pq = 1$ is of the following standard form :

   (a) I

   (b) II

   (c) III

   (d) IV

2. PDE $2r + 5s + 2t = 0$ has the solution :

   (a) $z = \phi_1(y - 2x) + \phi_2(2y - x)$

   (b) $z = \phi_1(x - 2y) + \phi_2(x + 2y)$

   (c) $z = \phi_1(2x + y) + \phi_2(2x - y)$

   (d) None of these

3. Heat equation is :

   (a) $\dfrac{\partial U}{\partial x} = c^2 \nabla^2 x^2$

(b) $\dfrac{\partial U}{\partial t} = c^2 \nabla^2 U$

(c) $\dfrac{\partial U}{\partial y} = \dfrac{1}{c^2} \nabla^2 t^2$

(d) $\dfrac{\partial U}{\partial t} = c^2 \nabla^2 y^2$

4. If U is independent of z in Laplace's equation in cylindrical co-ordinates, it reduces to :

(a) Green's equation

(b) Laplace's equation in Polar co-ordinates

(c) Gauss's equation

(d) Poisson's equation

5. PDE $\dfrac{\partial U}{\partial t} = k \dfrac{\partial^2 U}{\partial x^2}$ is :

(a) An elliptic differential equation

(b) A parabolic differential equation

(c) A circular differential equation

(d) None of these

6. $\int_0^2 e^{4t} \delta(2t-3)\, dt$:

(a) e^6

(b) $\dfrac{e^6}{2}$

(c) $\dfrac{e^6}{4}$

(d) $\dfrac{e^6}{6}$

7. If U is harmonic in D, then $\int_D \nabla^2 U \, dV =$

(a) $\int_{\partial D} \frac{\partial U}{\partial n} dS$

(b) $\int_{\partial D} \frac{\partial V}{\partial S} dS$

(c) $\int_{\partial D} \frac{\partial U}{\partial V} dS$

(d) None of these

8. Vibrations of a circular membrane are governed by :

(a) One-dimensional wave equation

(b) Two-dimensional wave equation

(c) Three-dimensional wave equation

(d) None of these

Section—B

9. Eliminate the arbitrary functions f and g from $y = f(x - at) + g(x + at)$.

10. Solve :

$$xr + p = 9x^2y^3$$

11. Solve the boundary value problem $\frac{\partial U}{\partial x} = 4\frac{\partial U}{\partial y}$ with $U(0, y) = 8e^{-3y}$ by the method of separation of variables.

12. If an ordinary function $f(x)$ has a simple zeros at $x = a$, then show that :

$$\delta\{f(x)\} = \frac{\delta(x - a)}{|f(a)|}$$

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13. Solve :

$$(D^2 + DD' + D' - 1)z = \sin(x + 2y)$$

14. Solve :

$$p^2 + q^2 = x + y$$

Section—C

15. Prove that the average value of a harmonic function is a sphere equal to its value at the centre.
16. By using method of separation of variable solve one-dimensional wave equation :

$$\frac{\partial^2 U}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2}$$

17. Find the steady state temperature distribution in a rectangular plate of sides a and b insulated at the lateral surface and satisfying the boundary conditions :

$$U(0, y) = U(a, y) = 0 \quad , \quad 0 \le y \le b$$

and $\quad U(x, 0) = 0,\ U(x, b) = f(x), \quad 0 \le x \le a$

18. Find the surface passing through two lines :

$$z = x = 0$$

$$z - 1 = x - y = 0$$

satisfying $r - 4s + 4t = 0$.

Section—D

19. Find the Green's function for the first quadrant $0 \le x < \infty, \quad 0 \le y < \infty$. Also solve the boundary value problem $\nabla^2 U = 0 = \dfrac{\partial^2 U}{\partial x^2} + \dfrac{\partial^2 U}{\partial y^2}$ with boundary conditions $U(x, 0) = f(x)$, $U(0, y) = 0$, $U \to 0$ as $x^2 + y^2 \to \infty$.

20. A string is stretched to two fixed points distance l apart. Motion is started by displacing the string in the form $U = a\sin\left(\frac{\pi x}{l}\right)$ from which it is released at time $t = 0$. Show that the displacement at any point at a distance x from one end at time t is given by :

$$U(x,t) = a\sin\left(\frac{\pi x}{l}\right).\cos\left(\frac{\pi ct}{l}\right)$$

21. Solve equation $r = t$ by Monge's method.

22. Solve :

$$x^2r - y^2t + xp - yq = \log x$$

Section—E

23. Find the steady state temperature distribution in a thin rectangular plate bounded by the lines $x = 0$, $x = a$, $y = 0$, $y = b$. The edges $x = 0$, $x = a$, $y = 0$ are kept at temperature 0 while the edge $y = b$ kept at 100°C.

24. State and prove Uniqueness theorem for the wave equation.